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Attention must next be turned to

$$\frac{1-\cos x}{x} \equiv u.$$

Writing c in place of $1 - \cos x$ the general equation is found at once to be

$$\frac{d^n u}{dx^n} + \frac{n}{x}\cdot\frac{d^{n-1}u}{dx^{n-1}} = \frac{1}{x}\cdot\frac{d^n c}{dx^n}.$$

Using precisely the same multipliers and method as before, we obtain

$$\frac{d^n u}{dx^n} = \mp \frac{1}{x}\cdot F_1(x)\cdot\cos x \pm \frac{n}{x^2}\cdot F_2(x)\cdot\sin x + \frac{\underline{|n}}{x^{n+1}}, \tag{A'}$$

$$\frac{d^n u}{dx^n} = \mp \frac{1}{x}\cdot F_3(x)\cdot\sin x \mp \frac{n}{x^2}\cdot F_4(x)\cdot\cos x - \frac{\underline{|n}}{x^{n+1}}. \tag{B'}$$

When n is even, (A'), and x is given pairs of values $+a$ and $-a$, $d^n u/dx^n$ assumes values that are equal in magnitude but opposite in sign. On the other hand, when n is odd this derivative alters neither in sign nor in magnitude when x is changed from $+a$ to $-a$. Hence, when n is odd $y = d^n u/dx^n$ is symmetrical with respect to the axis of ordinates.

Since all the separate terms of formulæ (A') and (B') involve either $\sin x$, or $\cos x$, or $\underline{|n}$ in the numerator and some positive integral power of x in the denominator we see that $d^n u/dx^n$ is finite for all values of x between 0 and ∞. When x becomes infinite $d^n u/dx^n = 0$. When $x = 0$ the indeterminate form 0/0, when treated in the usual way, gives $\pm \frac{x^n \sin x}{(n+1)x^n}$ for n even, and $\mp \frac{x^n \cos x}{(n+1)x^n}$ for n odd. Therefore, for $x = 0$, the nth derivative of u is equal to 0 or $\mp \frac{1}{n+1}$ according as n is even or odd, respectively.

For sake of brevity and variety, the question of the existence of maxima and minima may be settled by making use of the elementary properties of plane curves. The fraction c/x is finite, continuous, single-valued, with an infinite number of maxima and minima. Whenever u has a maximum or a minimum du/dx vanishes. In other words, du/dx changes sign for every value of x that corresponds to a maximum or a minimum of u. Also the general formulæ (A') and (B') show that any derivative is finite, continuous, and single-valued. Therefore, du/dx must likewise have an infinite number of maxima and minima. We then apply the foregoing argument to du/dx and d^2u/dx^2, and so on indefinitely. Obviously the same line of reasoning could have been applied to r and all of its x-derivatives. The earlier proof was presented *in extenso* because of its rigor and probable instructive value.

Proceeding as in the case of r, we find that, when $d^{n+1}u/dx^{n+1} = 0$,

$$\frac{d^n u}{dx^n} = \frac{1}{n+1}\cdot\frac{d^{n+1}c}{dx^{n+1}}.$$

Accordingly, at a maximum or minimum,

$$\left|\frac{d^n u}{dx^n}\right| \text{ equals } \frac{1}{n+1}\cdot|\sin x| \qquad \text{or} \qquad \frac{1}{n+1}\cdot|\cos x|$$

according as n is even or odd. Finally we conclude that

$$\left|\frac{d^n u}{dx^n}\right|_{x=0} = \frac{1}{n+1}, \text{ for } n \text{ odd},$$

$$\left|\frac{d^n u}{dx^n}\right| < \frac{1}{n+1}, \text{ for } n \text{ even, or for } n \text{ odd and } x \neq 0.$$

2732 [1918, 444]. Proposed by PAUL CAPRON, U. S. Naval Academy.

A conical cup, filled with fluid, stands with the vertex upward on a smooth horizontal surface. The inner and outer surfaces of the cup are similar cones of revolution, having altitudes h and $h(1 + x)$; the ratio of the specific weights of the material of the cone and the fluid is σ; the height of a barometer column of the fluid is h_0. Show that for equilibrium

$$\frac{h_0}{h}(1+x)^2 + \sigma x(1 + x + x^2/3) \not< 2/3.$$

SOLUTION BY THE PROPOSER.

Let w be the specific weight of the fluid; then the atmospheric pressure is wh_0. Let a be the inner radius of the cone and P = the vertical component of the pressure between the fluid and the cone.

The weight of the fluid is $\frac{1}{3}\pi a^2hw$, and
The weight of the cone is $\frac{1}{3}\pi a^2h\sigma w[(1 + x)^3 - 1]$.
The pressure between the horizontal surface and the cone is πa^2hw.
The atmospheric pressure upon the cone has the vertical component $\pi a^2(1 + x)^2wh_0$.
From the equilibrium of the fluid,

$$P + \tfrac{1}{3}\pi a^2hw = \pi a^2hw.$$

From the equilibrium of the cone,

$$\tfrac{1}{3}\pi a^2h\sigma w[(1 + x)^3 - 1] + \pi a^2(1 + x)^2wh_0 \nless P = \tfrac{2}{3}\pi a^2hw.$$

Hence, for equilibrium,

$$\frac{h_0}{h}(1 + x)^2 + \sigma x(1 + x + x^2/3) \nless 2/3.$$

2733 [1918, 444]. Proposed by J. L. RILEY, Stephenville, Texas.

An ellipse of constant eccentricity passes through the focus of a parabola and has its foci on the curve. Find the envelopes of its axes.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let F be the focus, $(a, 0)$ of the parabola,

$$y^2 = 4ax; \tag{1}$$

F_1, F_2, the foci, (x_1, y_1), (x_2, y_2) of the ellipse in any one of its positions; $2A$, e, the major axis and constant eccentricity.

We have

$$FF_1 + FF_2 = 2A, \tag{2}$$

$FF_1 = x_1 + a$, and $FF_2 = x_2 + a$; and hence

$$2a + x_1 + x_2 = 2A. \tag{3}$$

Let the equation of the major axis of the ellipse be

$$y = mx + b. \tag{4}$$

Eliminate y from (4) and (1), and we have the quadratic giving the abscissas x_1, x_2 of F_1, F_2,

$$m^2x^2 - (4a - 2bm)x + b^2 = 0 \tag{5}$$

and so

$$x_1 + x_2 = (4a - 2bm)/m^2. \tag{6}$$

Similarly eliminating x,

$$my^2 - 4ay + 4ab = 0, \tag{7}$$

and then

$$y_1 + y_2 = 4a/m, \tag{8}$$

and (3) becomes

$$(2am^2 - 2bm + 4a)/m^2 = 2A. \tag{9}$$

Again,

$$(F_1F_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = 4A^2e^2. \tag{10}$$

From (5),

$$x_1x_2 = b^2/m^2, \tag{11}$$

and from (7),

$$y_1y_2 = 4ab/m. \tag{12}$$

From (6) and (11),

$$(x_1 - x_2)^2 = 16a(a - bm)/m^4, \tag{13}$$

and from (8) and (12).

$$(y_1 - y_2)^2 = 16a(a - bm)/m^2. \tag{14}$$